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Brief communication

Trajectory mechanism for particle deposition in annular flows

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1. Introduction

Two mechanisms for deposition have been proposed for gas–liquid annular flows. One of these involves a consideration of the turbulent motion of the drops (Cousins and Hewitt, 1968; Hutchinson et al., 1971) caused by fluid velocity fluctuations. Another, first described by Anderson and Russell (1970), involves a trajectory mechanism whereby drops move in unidirectional motion from one wall to another in vertical conduits. Some progress has been made in describing deposition by turbulence. However, the trajectory mechanism has received less attention.

James et al. (1980) considered photographic studies of trajectories of drops with diameters larger than 250 μm and concluded that gas phase turbulence was having a moderate effect. Andreussi and Azzopardi (1983) point out that, for air–water systems, a significant volume fraction of the drops have diameters less than 250 μm. They show that available data on deposition rates suggest that both turbulent and trajectory mechanisms need to be considered.

The present paper presents numerical simulations of the deposition process which provide quantitative criteria for determining when a trajectory mechanism will dominate equations for the rate of deposition at very low concentrations. Analyses of the trajectory mechanism, presented by Anderson and Russell (1970) and by Chang (1973), have assumed that the velocity of depositing particles is the same as the velocity with which they are ejected from the wall film into the gas flow. However, this can be in error because there is a transition region in which particles could slow down while traversing the field, due to fluid resistance and particle turbulence. This effect is quantified in the calculations.

2. Approach

The simplified model of annular flow described in previous papers from this laboratory (Mito and Hanratty, 2003, 2004a,b, 2005, 2006) is used. A vertical two-dimensional channel, having a width of $2H$, is

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considered. Cartesian coordinates x_1, x_2 and x_3 are assigned to the streamwise, wall-normal and spanwise directions. The walls are located at $x_2 = 0$ and $x_2 = 2H$ and are represented as being arrays of particle sources which inject spherical solid particles into the field with specified velocity components in the streamwise and wall-normal directions, V_1^{0+} and V_2^{0+}. The + superscript indicates that the terms have been made dimensionless using the wall parameters, v^* and ν/v^*, where v^* is the friction velocity and ν is the kinematic viscosity. Velocity V_2^{0+} was varied and V_1^{0+} was set equal to 15, the fluid velocity at the edge of the viscous wall layer. The wall sources are assumed to be distributed uniformly and continuously on the walls at all times. Then the concentration field is calculated by using a Lagrangian method, which pictures the field as resulting from instantaneous wall sources that propelled particles into the field at different previous times (Mito and Hanratty, 2003). (Periodicities are not used in the x_1 and x_3 directions in the calculation of the concentration field.) Particles were removed from the field when they struck a wall. Results are presented for a fully-developed condition where the rate of injection equals the rate of deposition.

The suspension is assumed to be dilute enough to ignore collisions and the feedback effect on the fluid turbulence. The theoretical problem is to describe the behavior of a wall source. The change of the location of a particle is related to the velocity V_i by

$$\frac{\mathrm{d}x_i^+}{\mathrm{d}t^+} = V_i^+ \tag{1}$$

and the change of the velocity is described by the momentum equation

$$\frac{\mathrm{d}V_i^+}{\mathrm{d}t^+} = -\frac{3\rho_\mathrm{f} C_\mathrm{D}}{4d_\mathrm{p}^+\rho_\mathrm{p}}|\boldsymbol{V}^+ - \boldsymbol{U}^+|(V_i^+ - U_i^+) + g_i^+, \tag{2}$$

where d_p is the particle diameter, ρ_p is the density of the particle, ρ_f is the density of the gas, g_i is a component of the acceleration of gravity and C_D is the drag coefficient given by

$$C_\mathrm{D} = \frac{24}{Re_\mathrm{p}}(1 + 0.15Re_\mathrm{p}^{0.687}). \tag{3}$$

The dimensionless inertial time constant is defined as

$$\tau_\mathrm{p}^+ = \frac{4d_\mathrm{p}^+(\rho_\mathrm{p}/\rho_\mathrm{f})}{3C_\mathrm{D}|\boldsymbol{V}^+ - \boldsymbol{U}^+|}. \tag{4}$$

Thus Eq. (2) can be rewritten as

$$\frac{\mathrm{d}V_i^+}{\mathrm{d}t^+} = -\frac{V_i^+ - U_i^+}{\tau_\mathrm{p}^+}, \tag{5}$$

where effects of g_i are ignored. For a Stokes law resistance

$$\tau_\mathrm{p}^+ = \frac{d_\mathrm{p}^{+2}(\rho_\mathrm{p}/\rho_\mathrm{f})}{18}. \tag{6}$$

In order to solve Eq. (2), it is necessary to specify the fluid velocity, U_i, seen by the particles. This is represented by a stochastic model that uses a modified Langevin equation (Mito and Hanratty, 2002).

The effects of particle inertia on the behavior of the system are represented by a volume-averaged time constant

$$\tau_\mathrm{pB}^+ = \frac{1}{2HC_\mathrm{B}}\int_0^{2H} \overline{\tau}_\mathrm{p}^+(x_2)\overline{C}(x_2)\mathrm{d}x_2, \tag{7}$$

where C_B is the bulk concentration defined as

$$C_\mathrm{B} = \frac{1}{2H}\int_0^{2H} \overline{C}(x_2)\mathrm{d}x_2. \tag{8}$$

The rates of injection per unit area at the two walls are R_{A1} and R_{A2}. For a vertical system $R_{A1} = R_{A2} = R_A$. For fully-developed conditions the rate of injection equals the rate of deposition so a deposition coefficient can be defined as

$$k^+_{DB} = \frac{R_A}{C_B v^*}. \tag{9}$$

3. Results

The deposition coefficient made dimensionless with v^*, k^+_{DB}, is plotted against the particle time constant in Fig. 1 for $Re_\tau = v^* H/v = 590$. The calculations were done for three injection velocities, $V_2^{0+} = 0.5, 1.0, 2.0$. The particle diameter was kept constant, $d^+_p = 0.368$.

For small inertial time constants deposition is controlled by particle turbulence, caused by fluid velocity fluctuations. The particles disengage from the turbulence and glide to the wall by "free-flight." As τ^+_{pB} increases the "free-flight" starts at a larger distance from the wall, so the particles have a larger velocity component in a direction normal to the wall because the fluid turbulence is larger. Thus k^+_{DB} and $\overline{V}^+_d$ increase with τ^+_{pB} at small τ^+_{pB}.

At large enough inertial time constants the particles start their "free-flight" where the root-mean-square of the x_2-component turbulent velocity, σ^+, is relatively constant. This lead to the suggestion by Hay et al. (1996), Hanratty et al. (2000) and Pan and Hanratty (2002) that, for this situation, $k^+_{DB} \cong \sigma^+_p/2\pi$ at the edge of the viscous wall layer, where σ^+_p is the dimensionless root-mean-square of the particle turbulence. This relation is plotted as a solid curve in Fig. 1. It is noted that k^+_{DB} roughly follows this curve for a range of τ^+_{pB}.

Since particles injected from the wall become engaged with fluid turbulence, the calculations for different V_2^{0+} are roughly the same at small and moderate τ^+_{pB}. However, at very large τ^+_{pB}, the inertia of the injected particles is large enough that the influence of the injection velocity is dominant. This is accompanied by a sharp increase in k^+_{DB} and an eventual leveling out to a constant, which increases with increasing V_2^{0+}.

Fig. 2 plots $\overline{V}^+_d$, the average velocity with which particles strike the wall. It is noted that the plateaus in Fig. 1 correspond to situations in which the particles arrive at the wall at the same velocity with which they enter the field. This creates the picture that particles move from one wall to another with the same velocity with which they enter the field.

The calculations presented in Figs. 1 and 2 kept d^+_p constant while varying τ^+_{pB}. This meant that ρ_p/ρ_f was varying. In order to check whether the changes in k^+_{DB} were influenced by changes in the density ratio, calculations were also done for a constant ρ_p/ρ_f; that is, d^+_p varied along with τ^+_{pB}. The results are presented in Fig. 3. It is noted that the calculations with a constant ρ_p/ρ_f are quite close to what is obtained keeping d^+_p constant.

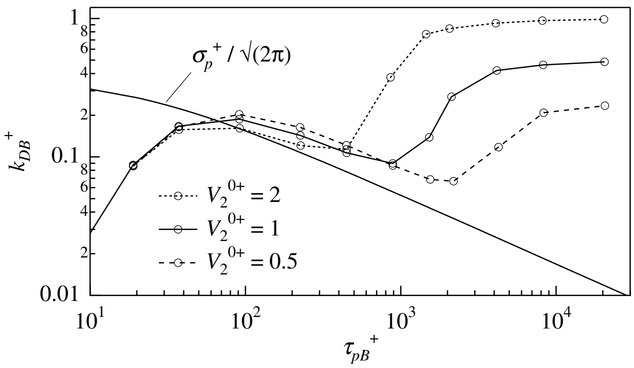

Fig. 1. Effect of the dimensionless inertial time constant, τ^+_{pB}, on the dimensionless deposition coefficient, k^+_{DB}, for $d^+_p = 0.368$ and $Re_\tau = 590$.

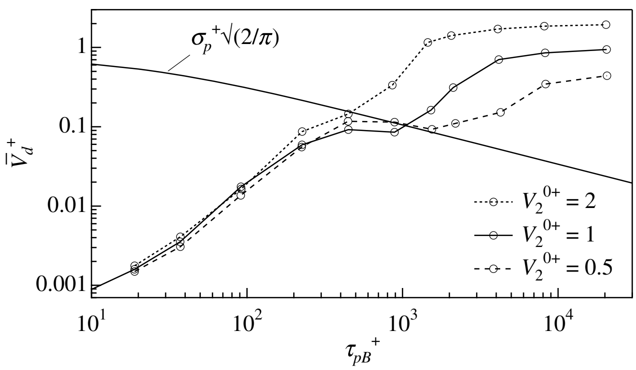

Fig. 2. Effect of the dimensionless inertial time constant, τ^+_{pB}, on the dimensionless mean velocity of particles striking the wall, $\overline{V}^+_{\mathrm{d}}$, for $d^+_{\mathrm{p}} = 0.368$ and $Re_\tau = 590$.

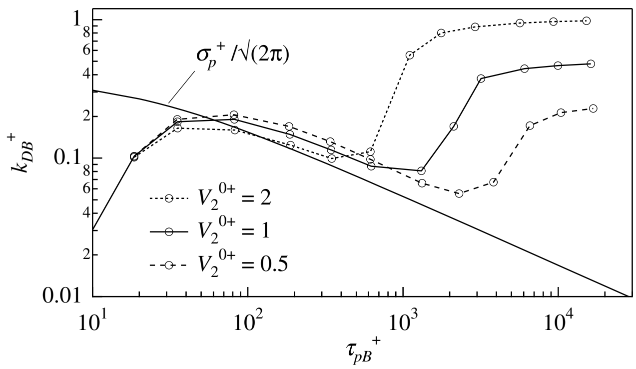

Fig. 3. Effect of the dimensionless inertial time constant, τ^+_{pB}, on the dimensionless deposition coefficient, k^+_{DB}, for $\rho_{\mathrm{p}}/\rho_{\mathrm{f}} = 1000$ and $Re_\tau = 590$.

The time it would take a particle with a velocity V^0_2 to move from wall to wall is $2H/V^0_2$ if fluid drag is ignored. Thus, a scaling which uses V^0_2 and $2H$ as the characteristic velocity and length scales would appear to be more appropriate than v^* and ν/v^* to correlate calculations of the deposition coefficient at large τ^+_{pB}.

This idea is tested in Fig. 4 where calculations are presented for three Reynolds numbers and three injection velocities. It is noted that all of the data fall on a single curve for $\tau_{\mathrm{pB}}V^0_2/2H$ greater than a number slightly

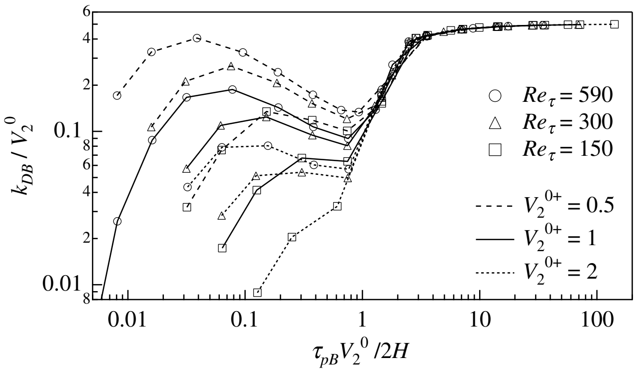

Fig. 4. Effect of the dimensionless inertial time constant, $\tau_{\mathrm{pB}}V^0_2/2H$, on the dimensionless deposition coefficient, k_{DB}/V^0_2, for $d^+_{\mathrm{p}} = 0.368$ and $Re_\tau = 590$.

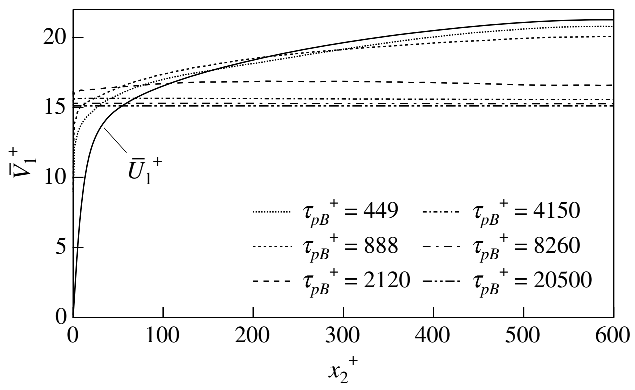

Fig. 5. Profiles of mean streamwise particle velocity for $d_p^+ = 0.368$ and $Re_\tau = 590$.

larger than one. At $\tau_{pB} V_2^0/2H$ greater than about 10 the deposition coefficient is constant. Thus, we can define the regime of $\tau_{pB} V_2^0/2H$ greater than 1.2 as the unidirectional trajectory regime. This is not surprising since the definition of τ_p is such that $V_2^0\tau_p$ is the stopping distance for a particle launched into a still fluid with velocity V_2^0.

The sharp decrease of k_{DB} with decreasing $\tau_{pB} V_2^0/2H$ in the trajectory region reflects both the deceleration of the particles before they hit the wall and the increasing influence of fluid turbulence.

Calculated mean velocities in the flow direction are compared with the velocity profile of the fluid, $\overline{U}_1^+(x_2^+)$, in Fig. 5. The particles are introduced with a streamwise velocity of $V_1^+ = 15$, the fluid velocity at the edge of the viscous wall layer. It is noted that for large inertial time constants the particle velocity is constant and approximately equal to the x_1-component of the injection velocity. (The particle concentration is also constant.) Over most of the cross-section the particles have a velocity which is less than the fluid. The significance of this is that the particles, on average, will exert a retarding force on the fluid. Recent work by Mito and Hanratty (2006) indicated that at volume fractions of about 10^{-4} the particles could significantly decrease the fluid turbulence. This would mean that the influence of fluid turbulence could be even less than indicated in the calculations presented in this paper.

4. Concluding remarks

The trajectory regime in a vertical conduit exists for $\tau_{pB} V_2^0/2H$ greater than 1.2. For $\tau_{pB} V_2^0/2H$ greater than ca. 10 the deposition coefficient, k_{DB}, is constant and the average velocity of depositing particles equals the velocity of the injected particles (as shown in Fig. 2). The rate of deposition per unit area, R_D, equals the product of the concentration of depositing particles and their velocity.

Only half of the particles at the wall are depositing. The other half are being injected into the field. Therefore, the concentration of depositing particles equals $C_B/2$. Since $R_D = C_B V_2^0/2$ the deposition coefficient equals $V_2^0/2$ (as shown in Fig. 1).

A transition region exists for $1.2 < \tau_{pB} V_2^0/2H <$ ca. 10. In this region the mean velocity with which the particles deposit is less than V_2^0 because the particles are influenced by fluid turbulence and particles moving in a uniform trajectory are slowed down because of fluid resistance.

It is of interest to note that the saltation regime, observed in a horizontal conduit, is similar to the trajectory regime where the particles injected from the bottom wall are not affected by fluid turbulence. The saltation regime is defined for $g^+ (=g\nu/v^*) \geqslant$ ca. 0.04 for $Re_\tau = 590$ by Mito and Hanratty (2004a), where g is the magnitude of the acceleration of gravity. Thus the criterion for the saltation regime is affected by gravity, g, and fluid turbulence, Re_τ. The peaks of the trajectories, which can be calculated from Eqs. (1) and (2), move closer to the bottom wall as g^+ increases. Assuming that τ_p is constant (=Stokesian) along a trajectory, the equations for the displacement and velocity of a particle in the saltation regime were derived by Mito and Hanratty (2004a) as

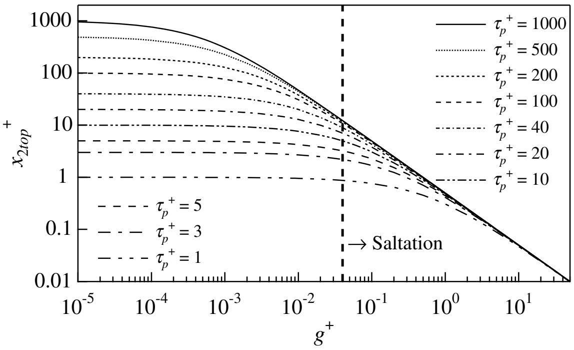

Fig. 6. Dimensionless height of trajectories of particles in the saltation region for $V_2^{0+} = 1$.

$$V_2 = (V_2^0 + g\tau_\mathrm{p})\exp\left(-\frac{t}{\tau_\mathrm{p}}\right) - g\tau_\mathrm{p}, \tag{10}$$

$$x_2 = \tau_\mathrm{p}\left[(V_2^0 + g\tau_\mathrm{p})\left\{1 - \exp\left(-\frac{t}{\tau_\mathrm{p}}\right)\right\} - gt\right]. \tag{11}$$

The height of the trajectory is calculated from Eqs. (10) and (11) as

$$x^+_{2\mathrm{top}} = \tau^+_\mathrm{p}\left[V_2^{0+} - g^+\tau^+_\mathrm{p}\ln\left(1 + \frac{V_2^{0+}}{g^+\tau^+_\mathrm{p}}\right)\right], \tag{12}$$

where $g\tau_\mathrm{p}$ equals the terminal velocity and all variables are made dimensionless using v^* and ν. Thus the height is a function of the wall-normal component of the injection velocity, the terminal velocity and the inertial time constant.

A plot of Eq. (12) for the case where $V_2^{0+} = 1$ is given in Fig. 6. The saltation region is indicated by the dashed curve. For $g^+ \to 0$ the results are the same as obtained for a vertical flow. For cases in which $x^+_{2\mathrm{top}}$ is greater than $2Re_\tau$, particles move in a unidirectional trajectory from the bottom wall to the top wall. It is noted that gravity has an important effect on the trajectory, in that $x^+_{2\mathrm{top}}$ decreases strongly with g^+. Thus, it is noted that it is difficult for non-turbulent trajectories to carry drops from the bottom wall to the top wall. The most likely scenario is that they are carried to a certain distance from the bottom wall where they either mix with the turbulence or drop back to the place of origin.

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